

# Indications of early thermalization in relativistic heavy-ion collisions

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The directed flow of particles emitted from the fireball created in a heavy-ion collision is shown to be a very sensitive measure of the pressure equilibration in the first 1 fm/c of the evolution. Performing a 3+1 dimensional (3+1D) relativistic hydrodynamic calculation with nonequilibrated longitudinal and transverse pressures, we show that the directed flow is strongly reduced if the pressure imbalance survives for even a short time. Transverse momentum spectra, elliptic flow and interferometry correlation radii are not very sensitive to this early pressure anisotropy. Comparison with the data points toward a short equilibration time of the order of 0.25 fm/c or less.

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## I. INTRODUCTION

Accumulated experimental observations from heavy-ion collisions at the BNL Relativistic Heavy Ion Collider indicate that a fireball of dense and hot matter is formed in the course of the collision [1]. The medium behaves as an almost perfect, thermalized fluid, with very small viscosity. Relativistic hydrodynamic models describe the experimental data for transverse momentum ( $p_\perp$ ) spectra, Hanbury Brown-Twiss (HBT) correlations radii and elliptic flow of particles [2–4]. Small deviations from local equilibrium due to large velocity gradients in the expansion have been quantified within the relativistic viscous hydrodynamics [5–15].

A separate, still unresolved question is how fast the initial, almost thermally equilibrated fluid is formed. If this time scale is very short ( $< 0.5$  fm/c), it would indicate that a strongly coupled system is formed in the collision. On the other hand, a longer equilibration time is compatible with weakly coupled perturbative QCD. Microscopic mechanisms responsible for the creation of the dense fireball are the subject of intense theoretical studies. Scenarios such as the development of instabilities of color fields, evolution of parton distributions in color fields, collisional equilibration, strong coupling solutions are proposed [16–21]. Models predict a larger value for the transverse  $P_\perp$  than the longitudinal  $P_\parallel$  pressure in the early stage. After some isotropization time  $\tau_{izo}$ , the two pressures become similar. The isotropization of the pressure is a necessary, but not a sufficient condition for equilibrium. However, in the following, we understand the equilibration time as the time when the pressure becomes (almost) isotropic, from that moment ideal (or more correctly, viscous) hydrodynamics applies. This means that we follow the natural assumption that the pressure equilibration is related to some microscopic processes driving the system toward equilibrium; in that case the time for the pressure isotropization can be used as an estimate for the equilibration time.

## II. NONISOTROPIC PRESSURE

The onset of the collective expansion in heavy-ion collisions can be separated into several stages. First, interacting, dense matter must be created after the initial nucleon collisions. Such an interacting system, can be effectively described using an energy-momentum tensor. For a system in equilibrium, the energy-momentum tensor has the form

$$T^{\mu\nu} = (\epsilon + P_{eq})u^\mu u^\nu - P_{eq}g^{\mu\nu}, \quad (1)$$

where  $u^\mu$  the fluid velocity,  $\epsilon$  the energy density, and  $P_{eq}$  the pressure. The creation of the interacting matter, close to equilibrium, does not mean that the elementary degrees of freedom are quasiparticles. The strongly interacting plasma is an almost perfect fluid [13, 14, 22]. The small value of the shear viscosity indicates that the system is very far from the limit of a kinetic description [23–25]. When the nonequilibrium effects represent only a correction to the ideal fluid picture, the viscous hydrodynamics can be applied [5]. Second order viscous hydrodynamics applies if the product of the viscosity coefficient times the velocity gradients is relatively small and if the initial pressures are close to equilibrium. In this paper we study the dynamics in the early stage, when the second assumption is not valid.

There is no realistic model of the equilibration that could be applied to heavy-ion phenomenology. In a short-living and strongly interacting system, quasiparticles cannot be formed. This causes problems for the application of kinetic models to the early phase [19, 26]. Field theory approaches use simplified models or geometries [17, 18, 20, 21, 27]. Formally, relativistic viscous hydrodynamics breaks down at short time scales, as stress corrections to the energy-momentum tensor are dominant. In this paper, we do not develop a microscopic model of the early evolution. Instead, we focus on possible observable consequences of the early nonequilibrium evolution. We assume that the nonequilibrium effects manifest themselves in the energy-momentum tensor as

anisotropic pressure

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_{eq} + \pi/2 & 0 & 0 \\ 0 & 0 & P_{eq} + \pi/2 & 0 \\ 0 & 0 & 0 & P_{eq} - \pi \end{pmatrix}. \quad (2)$$

In the early evolution, one expects that the transverse pressure is larger and the longitudinal pressure is smaller than the equilibrium one  $P_{eq}$ . This form of the energy-momentum tensor appears in the viscous hydrodynamics with Bjorken flow [6]. Similar correction to the longitudinal and transverse pressures are expected in a solution of strongly interacting systems or in the presence of classical color fields [20, 21, 28]. If a theory could be applied to the far from equilibrium evolution, the correction to the pressure  $\pi$  would be a solution of dynamical equations. Our goal is to study the effects of the possible presence of the nonequilibrium correction on the fluid dynamics and not to calculate the correction from an underlying theory. For that purpose it is sufficient to assume a time dependence of the stress correction  $\pi$  that describes the approach to equilibrium and use it to calculate its effect on observables in heavy ion collisions. The early equilibration means that stress corrections decrease with time, reaching zero at equilibrium. In a more realistic scenario the stress correction is not vanishing at large times, but should approach the value given by the shear viscosity effects. We do not take this into account in this first estimate, as we are interested in the very early stage of the dynamics.

The time scale when the pressure anisotropy decays is an estimate of the isotropization time, which for the bulk dynamics is equivalent to the thermalization time. A question of primary importance is the possibility of getting an experimental estimate of the equilibration time. Promising observables sensitive to the early dynamics are the dilepton and photon emissions [29]. The difficulties of such calculations are the uncertainties in the higher order corrections to the emission rates and a large background from other sources of dileptons and photons. Other possibilities [26, 30, 31] that have been explored study the effects of early off-equilibrium effects in the energy-momentum tensor on the hydrodynamic evolution.

Observables, such as  $p_\perp$  spectra, elliptic flow, or HBT radii are sensitive to the transverse flow profile at the freeze-out. The transverse flow is built up from the acceleration in the transverse direction, during the whole hydrodynamic evolution. In a boost invariant geometry the reduced longitudinal pressure acts only by changing the cooling rate. After fixing the final entropy per unit rapidity in all the scenarios, involving different longitudinal pressures, one obtains similar results [28, 32]. In the following we show, in a realistic 3 + 1D hydrodynamic simulation, that the  $p_\perp$  spectra, HBT radii, and elliptic flow are indeed not sensitive to the early pressure anisotropy.

One could argue that the longitudinal expansion itself

is sensitive to the reduced longitudinal pressure. However, the experimentally observed rapidity distributions can be reproduced by many scenarios with reduced  $P_\parallel$ , simply using different initial conditions [33]. With no a priori knowledge about the initial energy density, no conclusions can be drawn about how much and for how long the longitudinal pressure is reduced.

The initial energy density  $\epsilon(\eta_\parallel, x, y)$  for the 3 + 1D hydrodynamic evolution in the space-time rapidity  $\eta_\parallel$  and the transverse plane  $(x, y)$  is taken in the form

$$\epsilon \propto \left[ \left( \frac{(\eta_m + \eta_\parallel)N_+ + (\eta_m - \eta_\parallel)N_-}{\eta_m(N_+ + N_-)} \right) \frac{1 - \alpha}{2} \rho_{part} + \alpha \rho_{bin} \right] f(\eta_\parallel) \quad (3)$$

where the density in the transverse plane is proportional to a combination of participant nucleon  $\rho_{part} = N_+ + N_-$  and binary collision  $\rho_{bin}$  densities. On the right hand side of Eq. (3), only the dependence on  $\eta_\parallel$  is shown explicitly. The dependence on the position in the transverse plane comes from the densities of the right and left going participant nucleons  $N_\pm(x, y)$ , calculated from the Glauber model. The parameters and the longitudinal profile  $f(\eta_\parallel)$  are adjusted to reproduce the experimental spectra [4].

The factor

$$\frac{(\eta_m + \eta_\parallel)N_+ + (\eta_m - \eta_\parallel)N_-}{\eta_m(N_+ + N_-)} \quad (4)$$

in the initial energy density (3) is based on the assumption that forward going participant nucleons with density  $N_+$  emit particles predominantly in the forward hemisphere, with the reverse for the backward going particles. Such a distribution is expected in bremsstrahlung emission, and has been extracted from the analysis of the data in asymmetric collisions [34]. The beam rapidity  $\eta_m = \ln(\sqrt{s}/m_p)$  is taken as the rapidity of the emitting charge. The effect of this modification of the initial density is to cause a tilt of the initial source away from the collision axis (see Fig. 3 in [35]). In (3) only the density of participant nucleons is tilted. If we assume that the density of binary collisions is tilted as well, we have

$$\epsilon \propto \left[ \left( \frac{(\eta_m + \eta_\parallel)N_+ + (\eta_m - \eta_\parallel)N_-}{\eta_m(N_+ + N_-)} \right) \left( \frac{1 - \alpha}{2} \rho_{part} + \alpha \rho_{bin} \right) \right] f(\eta_\parallel), \quad (5)$$

which we call large tilt initial conditions. The variation in the magnitude of the initial tilt between formulas (3) and (5) is a measure of the uncertainty of the model.

Using the tilted initial conditions, the coefficient  $v_1$  of the directed flow [36] of particles emitted in Au-Au collisions at  $\sqrt{s} = 200\text{GeV}$  [35] has been described within the ideal fluid hydrodynamic model. Two important characteristics of this mechanism are of relevance for the study of the early isotropization. First, the directed flow is generated very early in the dynamics, mainly in the first

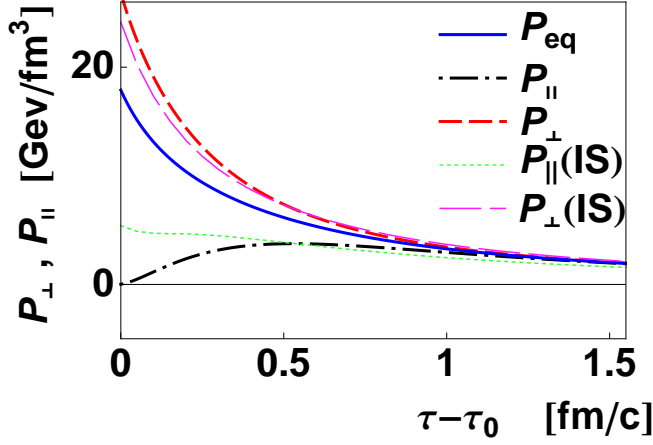


FIG. 1. (Color online) Time evolution of the longitudinal, transverse and equilibrium pressures (dash-dotted, dashed and solid lines respectively) at the center of the fireball in a central Au-Au collision, with  $P_L(\tau_0) = 0$  and  $\tau_{iso} = 0.25 \text{ fm/c}$ . The long-dashed and dotted lines represent the transverse and longitudinal pressures in viscous hydrodynamics with  $\eta/s = 1/4\pi$ .

1 fm/c. Second, the formation of the directed flow requires a simultaneous acceleration of fluid elements by the transverse and longitudinal pressures [35].

The mechanism generating the directed collective flow from a tilted initial source can be understood when considering the acceleration equations of relativistic hydrodynamics. For small initial times (small transverse velocities  $v_x, v_y$ ) the accelerations in the transverse  $x$  and longitudinal  $\eta_{||}$  directions take the form

$$\partial_\tau v_x = -\frac{\partial_x P_\perp}{\epsilon + P}, \quad (6)$$

$$\partial_\tau Y = -\frac{\partial_{\eta_{||}} P_{||}}{\tau(\epsilon + P)}, \quad (7)$$

where  $Y = \frac{1}{2} \ln((1 + v_z)(1 - v_z))$  is the rapidity of the fluid element. At freeze-out, a fluid element of rapidity  $Y$  and velocity  $v_x$  emits particles with a thermal smearing around these velocities. A negative directed flow  $v_1 < 0$  for positive pseudorapidities  $\eta > 0$ , means that most of the emission happens in fluid elements with negative correlation between  $v_x$  and  $Y$ . In a tilted source the early acceleration of the fluid occurs predominantly with anticorrelated signs in the  $v_x$  (6) and  $Y$  (7) directions. If in the early stage the longitudinal pressure  $P_{||}$  is significantly reduced, the directed flow is not generated. The sensitivity of the directed flow to the simultaneous action of the transverse and longitudinal pressures, makes it a preferred observable to measure the degree of pressure anisotropy.

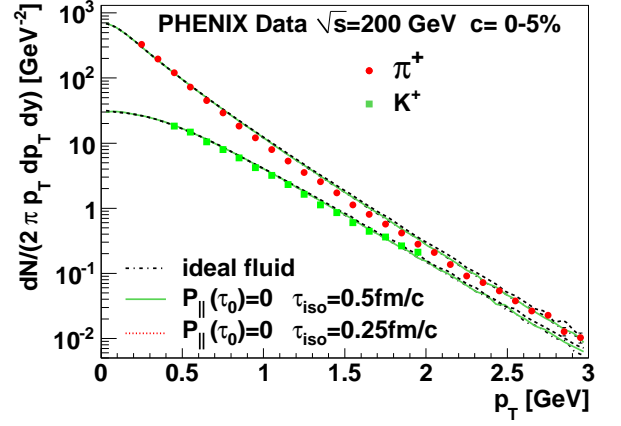


FIG. 2. (Color online) Transverse momentum spectra for  $\pi^+$  and  $K^+$ . The dash-dotted, solid and dotted lines represent the hydrodynamic results with  $\tau_{iso} = 0, 0.25, \text{ and } 0.5 \text{ fm/c}$  respectively; data are from the PHENIX Collaboration [37].

### III. RESULTS

We perform numerical simulations of the 3 + 1D hydrodynamics with anisotropic pressures. A phenomenological correction is added (2) to the ideal fluid energy-momentum tensor with

$$\pi(\tau, x, y, \eta_{||}) = P_{eq}(\tau_0, x, y, \eta_{||}) e^{(\tau_0 - \tau)/\tau_{iso}}. \quad (8)$$

The pressure anisotropy makes the longitudinal pressure  $P_{||} = P_{eq} - \pi$  zero initially, and the anisotropy decays exponentially with a relaxation time  $\tau_{iso}$ . A similar ansatz has been used in boost invariant calculation of the effect of the initial dissipation on the transverse expansion [30]. Other parameterizations of the form of the initial pressure anisotropy have been used [29, 31]. The goal of this paper is to propose a generic signature of the presence of the early anisotropy of the pressure, that can be used for any time dependence of the stress tensor correction, e.g. from microscopic models, as long as the far from equilibrium dynamics in the microscopic theory would imply, in the hydrodynamic limit, a pressure anisotropy.

The actual form and time dependence of the pressure anisotropy are not known from first principles. One expects that the large initial anisotropy decays and becomes small in the latter stages of the collision, as the shear viscosity coefficient of the matter formed in nuclear collisions is estimated to be small [11, 12]. Similar relaxation time terms appear in the second order relativistic viscous hydrodynamics [5], but the present investigation does not rely on the assumption of viscous hydrodynamics, which is not applicable at this very early stage, where the dissipative correction to the pressure is of the same order as the pressure itself.

The assumed form (8) is a phenomenological ansatz used to test the sensitivity of different observables on the early pressure anisotropy. The time scale in this parameterization is given by the isotropization time  $\tau_{iso}$ .

Another parameter is the initial value of the pressure anisotropy at the beginning of the collective expansion. In Eq. 8 we take  $\pi(\tau_0) = P_{eq}(\tau_0)$ , which gives zero longitudinal pressure at the beginning. The initial pressure is even less constrained than the time dependence of  $\pi$ , as it is defined by the energy deposition processes in the collision before the formation of the dense fireball. We have checked that qualitatively similar results are obtained, if the initial longitudinal pressure is varied between  $P_L(\tau_0) = P_{eq}/2$  and  $P_L(\tau_0) = -P_{eq}/2$ . Initial conditions with zero or negative initial longitudinal pres-

sure cannot be described in a kinetic theory or as viscous corrections to the ideal hydrodynamics.

The hydrodynamic equation are obtained boosting the energy-momentum tensor (2) by the fluid velocity  $u^\mu$

$$T^{\mu\nu} = (\epsilon + P_{eq} + \frac{\pi}{2})u^\mu u^\nu - (P_{eq} + \frac{\pi}{2})g^{\mu\nu} - \frac{3\pi}{2}v^\mu v^\nu, \quad (9)$$

with

$$u^\mu = (\gamma \cosh Y, u_x, u_y, \gamma \sinh Y), \quad (10)$$

and

$$v^\mu = \left( \gamma \sinh Y, \frac{\sinh Y u_x (\gamma^2 \cosh Y - \gamma)}{\sinh Y + \cosh Y (u_x^2 + u_y^2)}, \frac{\sinh Y u_y (\gamma^2 \cosh Y - \gamma)}{\sinh Y + \cosh Y (u_x^2 + u_y^2)}, \frac{u_x^2 + u_y^2 + \sinh^2 Y \cosh Y \gamma^3}{\sinh Y + \cosh Y (u_x^2 + u_y^2)} \right), \quad (11)$$

where  $\gamma = \sqrt{1 + u_x^2 + u_y^2}$ ,  $u_\mu v^\mu = 0$ . The resulting long hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \quad (12)$$

are not presented explicitly. We solve them numerically with initial conditions with the two different initial tilts of the source (3) and (5). The initial time  $\tau_0 = 0.25 \text{ fm}/c$  is chosen. The small value for the time at which the transverse expansion begins is favored in order to reproduce the experimental HBT data [3]. At the freeze-out temperature of 150 MeV, particles are emitted and resonance decays are performed using the event generator THERMINATOR [38]. In a kinetic picture of the nonequilibrium pressure, the momentum distributions of particles have nonequilibrium corrections [6]. Those modifications of the momentum distribution should be taken into account in the Cooper-Frye formula at the freeze-out. Such a kinetic picture of the dense matter at the very early stage is questionable, and is not used in the present investigation. The integration of the particle emission over the freeze-out hypersurface is taken for  $\tau \geq 1 \text{ fm}/c$ . In our calculation, the nonequilibrium correction are reduced after that time and the equilibrium distributions are used in the Cooper-Frye formula. We have checked that including the emission, using equilibrium distributions, for  $\tau < 1 \text{ fm}/c$  modifies the results by less than 3%. In simulations with an initial pressure anisotropy, entropy is generated. The initial density must be rescaled to take this effect into account. The relative equilibrium entropy production is 29% and 61% for  $\tau_{iso} = 0.25$  and  $0.5 \text{ fm}/c$  respectively, very close to the value estimated from a formula valid in the Bjorken scaling expansion [30].

In Fig. 1 is shown the time dependence of the longitudinal and transverse pressures. The pressures equilibrate fast to the isotropic value. Figs. 2 and 3 show results for the transverse momentum spectra and HBT radii for Au-Au collisions of centrality 0–5%. For the three cases

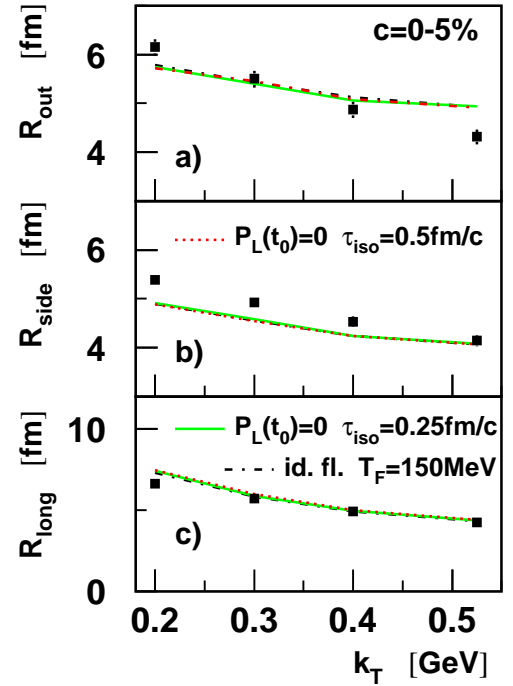


FIG. 3. (Color online) STAR Collaboration data [39] (squares) for the HBT radii ( $R_{out}$ ,  $R_{side}$  and  $R_{long}$  in panels a), b) and c) respectively) compared to the results of hydrodynamic simulations (same lines as in Fig. 2).

considered : isotropization with  $\tau_{iso} = 0$  (ideal fluid), 0.25 and  $0.5 \text{ fm}/c$ , the  $p_\perp$  spectra and the HBT radii are indistinguishable. This confirms the existence of a universality in the transverse flow for different longitudinal pressures [28]. The reduced longitudinal pressure in the isotropization scenario, implies a slower cooling, but renormalizing the initial conditions to obtain the same final multiplicity cancels this effect. The source lives for

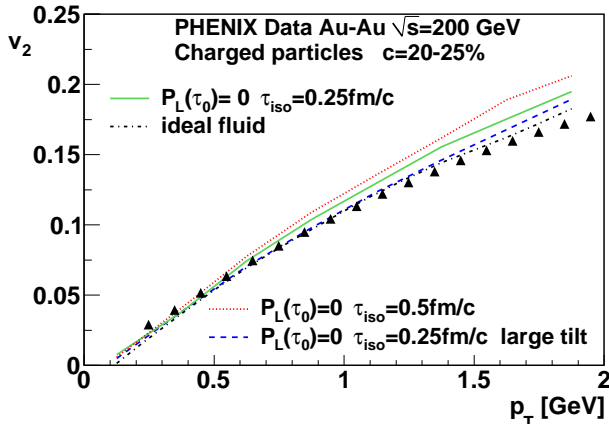


FIG. 4. (Color online) The elliptic flow coefficient  $v_2$  of charged particles from the PHENIX Collaboration [40] (triangles) for centrality 20 – 25% compared to hydrodynamic calculations; the dashed line is for  $\tau_{iso} = 0.25\text{fm}/c$  and the maximal tilt of the source (Eq. 5) and other lines are as in Fig. 2.

the same total time and the same transverse flow is generated in all the cases considered.

For semi-central collisions azimuthally asymmetric emission can be tested. The elliptic flow coefficient  $v_2$  (Fig. 4) is very similar for the different isotropization times tested. Again, we find that the elliptic flow is not an observable sensitive to the early pressure anisotropy.

On the other hand, the directed flow varies very much depending on the longitudinal pressure active in the early expansion. Fig. 5 presents  $v_1$  for charged particles as a function of pseudorapidity. When the longitudinal pressure is reduced, the directed flow is not generated with enough strength. It demonstrates that the longitudinal acceleration must be active (Eq. 7) very early to generate enough directed flow.

To estimate the thermalization time  $\tau_{iso}$ , the initial deformation of the source must be known. We use two extreme assumptions for the value of the tilt. For the smaller tilt (Eq. 3), the experimental data is described using  $\tau_{iso} = 0$  (ideal fluid). The expansion of the source with the larger tilt (Eq. 5) is compatible with the data if the longitudinal pressure is retarded by  $0.25\text{fm}/c$  with respect to the transverse pressure. The conclusion of this analysis is that the isotropization time of the pressure is smaller than  $0.25\text{fm}/c$ . This time can be treated as an effective thermalization time of the medium.

The question arises whether such a small degree of pressure anisotropy is compatible with viscous corrections to the ideal fluid flow, which is the minimal mechanism generating nonequilibrium corrections to the energy-momentum tensor. Full 3 + 1D viscous hydrodynamic calculations of the directed flow are not yet available. For the Bjorken flow at the early stage, shear viscosity corrections have the same form as in Eq. (2) and the correction to the pressure  $\pi$  is the solution of a dynam-

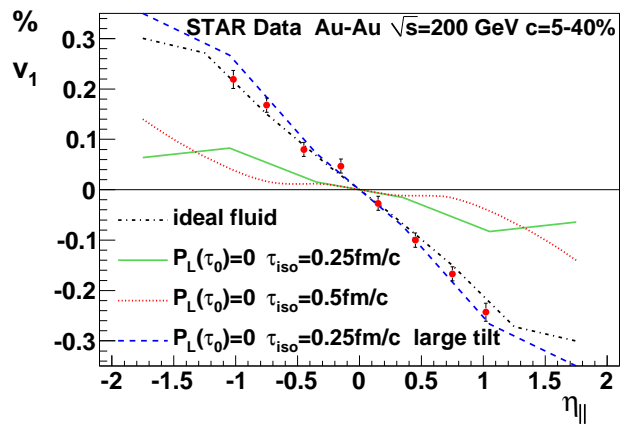


FIG. 5. (Color online) The directed flow from hydrodynamic calculations (same lines as in Fig. 4) compared to STAR Collaboration data [36].

ical equation [5]. In Fig. 1 are shown the longitudinal (dotted line) and transverse (long-dashed line) pressures resulting from the action of the shear viscosity, corresponding to the strong coupling limit ( $\eta/s = 1/4\pi$ ) [23], with the Navier-Stokes initial value of the stress correction  $\pi = 4\eta/3\tau_0$ . The pressure anisotropy from such a small shear viscosity is compatible with our limits on the pressure anisotropy. It indicates that the shear viscosity in the dense matter at the early stage of the collision is close to the strong coupling limit. A different issue is the role of shear and bulk viscosities in the latter expansion. In particular, the viscosity and dissipative effects in the hadronic rescattering are known to influence significantly the final elliptic flow [41].

#### IV. CONCLUSIONS

We propose to measure the thermalization time in the early stage of a heavy-ion collision using the directed flow of particles. We demonstrate in explicit hydrodynamic calculations that the directed flow is significantly reduced in the presence of even a very short pressure anisotropy. The directed flow observable is unique, as it is sensitive simultaneously to the transverse and longitudinal pressures. Moreover, the directed flow is generated early in the expansion. Hydrodynamic calculations indicate that observables such as the  $p_\perp$  spectra, HBT radii and elliptic flow are sensitive to the whole evolution of the fireball, and feel the action of the transverse flow only. A short reduction of the longitudinal pressure does not influence these transverse flow observables.

Using the initial fireball densities calculated in the Glauber model, we estimate that the thermalization time is smaller than  $0.25\text{fm}/c$ . Such a small value of the delay for the appearance of the longitudinal pressure indicates that the system is strongly coupled. For small deviations from equilibrium, the AdS/CFT result for the relaxation

time is  $\tau_\pi = (1 - \ln 2)/(6\pi T) \simeq 0.02 \text{ fm}/c$  [42], which is  $\simeq 30$  times smaller than the value for the massless Boltzmann gas. Our result also points toward a small shear viscosity in the dense plasma, as otherwise the longitudinal pressure would be significantly reduced. We note that the directed flow could serve as a sensitive constraint for microscopic models of the initial equilibration [15, 17–

21, 26, 27, 31]; with approaches based on field theory solutions [20, 21] or kinetic theory [19, 26] being more general than our parameterization, as they describe both the far from equilibrium dynamics and the near equilibrium viscous hydrodynamics.

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